K, n, rheological constants; ρ, density; g, acceleration of gravity; α_0, coefficient of surface tension; t, t_δ, dimensional and dimensionless times; Π_i, dimensionless complexes; v, v_{max}, velocity of lowering, maximal velocity of lowering; Ψ, Ψ_0, dimensionless functions.

LITERATURE CITED

1. Z. P. Shul'man and B. M. Khusid, Nonstationary Processes of Convective Transfer in Nonlinear Hereditary Media [in Russian], Minsk (1983).
2. Z. P. Shul'man, V. V. Mel'nichenko, B. M. Khusid, A. A. Dunets and O. I. Mikulin, "Motion of an elastic viscous liquid in impact with an obstacle and spreading on the horizontal surface under the gravitational force," Preprint, ITMO AN BSSR, No. 13, Minsk (1985).
3. B. D. Summ and Yu. V. Goryunov, Physicochemical Foundations of Wetting and Spreading [in Russian], Moscow (1976).
4. L. I. Sedov, Methods of Similarity and Dimensionality in Mechanics [in Russian], Moscow (1972).
5. V. A. Zhuzhikov, "Dimensionless relationships and dimensional analysis," Dep. at VINITI, 14.07.75, No. 2492-75, Moscow (1975).
6. V. A. Ivanov, Modeling the Processes of Obtaining and Processing Polymeric Materials [in Russian] (1985), pp. 102-109.
7. I. K. Berezin, A. M. Golubitskii, and A. I. Ponomarchuk, "Numerical methods for calculating flows of high-viscosity liquids with free surfaces," Preprint, Institute of Mechanics of Continuous Media, Ural Division of AN SSSR, Perm (1988).

EFFECTIVE VISCOPLASTICITY PARAMETERS OF SUSPENSIONS

V. A. Buryachenko

UDC 539.4:541.24

The effective field method is used to determine effective parameters of suspensions consisting of rigid ellipsoidal inclusions in a nonlinear viscoplastic matrix.

1. General Relationships. Within a macroregion z with characteristic function Z we will consider a suspension containing a statistically large number of rigid ellipsoidal inclusions and an incompressible viscoplastic matrix, the mechanical properties of which are described by a dissipative function

$$D = k\sqrt{\varepsilon_{ij}\varepsilon_{ij}} + \frac{1}{2}\eta(\varepsilon_{kl})(\varepsilon_{ij}\varepsilon_{ij}) + ae_{ij}\varepsilon_{ij}. \tag{1}$$

For definiteness, we will consider the variant of a power-law liquid $\eta(\varepsilon_{k\ell}) = \eta_0(I_2')^{(n-1)/2}$, where $I_2' = э_{k\ell}э_{k\ell}$ is the second invariant of the deformation rate deviator $э_{k\ell} = \varepsilon_{k\ell} - \varepsilon_{ii}/3$, $\varepsilon_{ii} = 0$ from the incompressibility condition.

The matrix contains a Poisson set $X = (V_k, x_k, a^i, \omega_k)$ of ellipsoidal rigid inclusions v_k with characteristic functions V_k, centers x_k forming a Poisson set, semiaxes a^i ($a^1 > a^2 > a^3$) and set of Euler angles ω_k with the inclusions having identical dimensions but various orientations. We will assume the random fields X, σ, ε, э ergodic and statistically homogeneous, so that averaging over the set can be replaced by averaging over characteristic volumes:

$$\langle(\cdot)\rangle_\alpha = \bar{v}_\alpha^{-1}\int(\cdot)V_\alpha(x)\,dx,\ \bar{v}_\alpha = \operatorname{mes} v_\alpha,\ \alpha = 0,\ 1,\ \ldots,$$

$$\langle(\cdot)\rangle = (\operatorname{mes} z)^{-1}\int(\cdot)Z(x)\,dx,$$

$V_0 = Z - V$, $V = \sum_k V_k$. In the future we will use the notation $\langle\cdot \mid x_1; x_2\rangle$ for the conditional average over the set X, where at x_1, x_2 we have inclusions with $x_1 \neq x_2$.

Moscow Institute of Chemical Machine Design. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 58, No. 3, pp. 452-456, March, 1990. Original article submitted November 10, 1988.

0022-0841/90/5803-0331$12.50 ©1990 Plenum Publishing Corporation

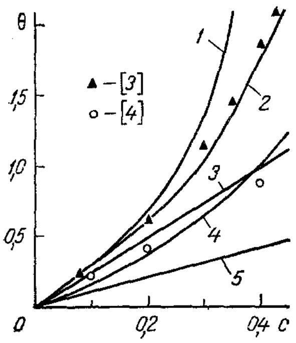

Fig. 1

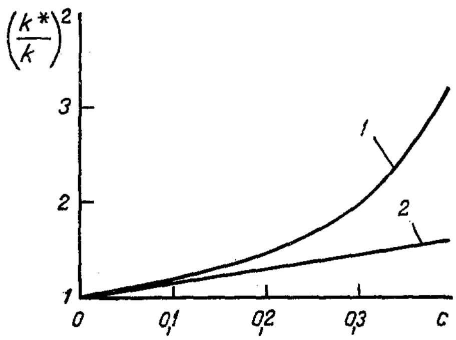

Fig. 2

Fig. 1. Experimental data and calculated curves of relative change in suspension viscosity.

Fig. 2. Calculated curves of relative change in suspension plasticity limit.

For Eq. (1) we have a local associative law of component flow

$$\sigma_{ij} = \frac{k\varepsilon_{ij}}{\sqrt{\varepsilon_{kl}\varepsilon_{kl}}} + \frac{n+1}{2}\eta(\varepsilon_{kl})\,\varepsilon_{ij} + ae_{ij}.$$

Without subscripts the flow law has the form

$$\sigma = L_0(\varepsilon + be), \tag{2}$$

where for the isotropic tensors L_0, b we use the notation:

$$L_0 = (3k_0,\ 2\mu_0) = 3k_0N_1 + 2\mu_0N_2,\ b = (3b_1,\ 2b_2),$$
$$N_{1ijkl} = \delta_{ij}\delta_{kl}/3,\ N_{2ijkl} = (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - 2\delta_{ij}\delta_{kl}/3)/2,$$
$$2\mu_0 = k(\varepsilon_{ij}\varepsilon_{ij})^{-\frac{1}{2}} + \frac{n+1}{2}\eta_0(\varepsilon_{ij}\varepsilon_{ij})^{\frac{n-1}{2}},\quad 2b_2 = a/2\mu_0.$$

In obtaining the final results we take the parameters k_0 and b_1 infinite. It is assumed that the matrix hydrodynamics are determined by the equations of creeping flow, and the only interaction between inclusions is hydrodynamic. Brownian motion will be neglected. Equation (2) is nonlinear, and in order to use the well known methods of the linear theory of elasticity [1, 2] to obtain effective rheological laws we must linearize that equation, having made the additional assumption L_0, be = const within the matrix. In doing this we have made the assumption of homogeneity of accumulated plastic deformation and the second invariant of the deformation rate tensor $I_{20} = \varepsilon_{ij}\varepsilon_{ij} = \langle\varepsilon_{ij}\varepsilon_{ij}\rangle_0$ within the matrix.

Introducing the modified deformation rate $\omega(x) = \varepsilon(x) - \alpha_0$, where $\alpha_0 = -be$, we obtain the equilibrium equation of the suspension

$$\nabla L_0 w = -\nabla\{[L]w - L_1\alpha_1\}, \tag{3}$$

where ∇ is the symmetrized gradient operation; $\alpha_1 = -\alpha_0 V(x)$; L_1 is the inclusion parameter, analogous to L_0; $[L] = L_1 - L_0$.

To the accuracy of the notation used, Eq. (3) coincides with the analogous expression of thermoelastic equilibrium of an inhomogeneous medium [1]. Therefore we can make use of the effective field method for its solution. We then use the fundamental solution G corresponding to Eq. (3) to transform the latter to an integral equation, centralizing which we obtain

$$w(x) = \langle w\rangle - \int U(x-y)\{[L]w - L_1\alpha_1 - [\langle [L]w_0\rangle - \langle L_1\alpha_1\rangle]\}\,dy, \tag{4}$$

where $U(x - y) = \nabla\nabla\cdot G(x - y)$.

To define the effective viscoplastic property tensor L^* and the macroscopic plastic deformation tensor α^* in the equation

$$\langle \sigma \rangle = L^* (\langle \varepsilon \rangle - \alpha^*) \tag{5}$$

we must evaluate the tensors A and H in the relationships $\langle \sigma V \rangle = A\langle \varepsilon \rangle$, $\langle \sigma V \rangle = H$ at $\alpha = 0$, $\sigma^0 = \langle \sigma \rangle \neq 0$ and $\alpha_0 \neq 0$, $\sigma^0 = 0$ respectively. Then

$$L^* = L_0 (I - A)^{-1}, \ \alpha^* = \langle \alpha_0 + \alpha_1 (x) \rangle - L_0^{-1} H. \tag{6}$$

2. Evaluation of L^*, α^*. We will specify the suspension structure by a binary distribution function $\varphi(v_m \mid v_k)$ — the probability of an inclusion distribution in the region v_m for fixed v_k. We assume that φ is centrally symmetric:

$$\varphi (v_m | v_k) = \psi (v_m)(1 - V'_{km}) n^c (\text{mes } z)^{-1}, \tag{7}$$

where n^c is the numerical inclusion concentration, related to the volume concentration $c = 4/3\, \pi a^1 a^2 a^3 n^c/3$; V_{km}' is the characteristic function of a sphere with center at x_k and radius $a_{km} = a^3 + a^1$; $\psi(\omega_m)$ is the density of the distribution ω_m, normalized to the condition $\langle \psi(\omega_m) \rangle = 1$.

We will define an arbitrary realization of the field X and consider the field $\overline{\sigma}(x)$ (which we will term the effective field below), $x \in v_k$, in which the inclusion is found:

$$\overline{\sigma} (x) = \sigma^0 + \int \Gamma (x - y)\{V (y;\ x)[- L_0^{-1} (y) \sigma (y) + \alpha_1 (y)] \ - [\langle - L_0^{-1} \sigma \rangle + \langle \alpha_1 \rangle]\}dy, \tag{8}$$

where $V(y; x) \equiv V(y) - V_k(x)$; $\Gamma(x-y) = -L_0(I\delta(x-y) + U(x-y)L_0)$ is Green's internal stress tensor. The field X and hence $\overline{\sigma}(x)$ are random; to find mean values of functions of $\overline{\sigma}(x)$ we introduce the effective field hypotheses [1, 2]:

H1) the field $\overline{\sigma}_k \equiv \overline{\sigma}(x_k)$ is homogeneous in the vicinity of each fixed inclusion v_k;

H2) each m (m > 1) inclusions are located in an inhomogeneous effective field $\hat{\sigma}_{1, \ldots, m}$, independent of the properties of the inclusion considered.

The homogeneous field $\overline{\sigma}_k$ uniquely defines the field within the inclusion

$$\sigma^+ = B_k (\overline{\sigma}_k - Q_k \alpha_1), \ B_k = (L_0 P_k)^{-1}, \ Q_k = L_0 (I - P_k L_0), \tag{9}$$

where the constant tensor $P_k = -\int U(x-y)V_k(y)dy\,(x \in v_k)$ is independent of the dimensions of the ellipsoid and can be expressed in a known manner in terms of the Eshelby tensor [1, 2].

Substituting Eq. (9) in Eq. (8), we find

$$\overline{\sigma} (x) = \sigma^0 - \int \Gamma (x - y)\{[L_0^{-1} B (y) \overline{\sigma} (y) + B (y) \alpha_1 (y)] V (y;\ x) \ - [\langle L_0^{-1} B\overline{\sigma} \rangle + \langle B\alpha_1 \rangle]\} \, dy. \tag{10}$$

We now average Eq. (10) over the set $X(\cdot \mid v_k)$ with the aid of Eq. (7):

$$\langle \overline{\sigma} (x) \rangle = \sigma^0 - \int \Gamma (x - y)\{ \langle [L_0^{-1} B (y) \overline{\sigma} (y) + B (y) \alpha_1 (y)] V (y;\ x)|y;\ x \rangle \ - [\langle L_0^{-1} B\overline{\sigma} \rangle + \langle B\alpha_1 \rangle]\} \, dy. \tag{11}$$

To calculate the arbitrary moments in Eq. (11) we make use of hypothesis H2 and the assumption $\overline{\sigma}_{12} = \overline{\sigma} =$ const. Solving the problem of binary interaction of point inclusions [1] with centers at x and y, located in an infinite matrix under the action of a field defined at infinity $\overline{\sigma}$, from Eq. (11) we find $\langle \overline{\sigma} \rangle$, whereupon from Eqs. (9), (6) we obtain [1, 2]

$$L^* = L_0 (I - C \langle BV \rangle)^{-1}, \ \alpha^* = L_0 / L^* \alpha_0,$$

$$C = (I + \langle Q \rangle L_0^{-1} - \int \Gamma (x - y) L_0^{-1} \langle BV \rangle \Gamma (x - y) L_0^{-1} \langle BV \rangle (1 - V'_{12} (y)) \, dy)^{-1}. \tag{12}$$

3. Evaluation of Medium Loading Function. The deformation properties of the medium can be described by the dissipative function of the entire macrovolume

$$\langle D \rangle = L^* (\langle \varepsilon \rangle - \alpha^*) \langle \varepsilon \rangle, \tag{13}$$

which in the case of rigid inclusions can be represented in the form

$$\langle D \rangle = (1 - c) \langle \sigma\varepsilon \rangle_0. \tag{14}$$

For rigid inclusions in an incompressible matrix the tensors B, D, L^*/L_0 are independent of the rheological properties of the matrix and are defined solely by the geometric microstructure of the medium. Then to obtain the explicit dependence of $\langle D\rangle$ on $\langle\varepsilon\rangle$ it is necessary to use Eqs. (13), (14) to evaluate the quantity I_{20}:

$$I_{20} = \langle \varepsilon_{ij}\varepsilon_{ij} \rangle = L_0^{-1}(\langle \sigma\varepsilon \rangle - L_0 b \langle e \rangle_0 \langle \varepsilon \rangle_0) = \\ = (1-c)^{-1}(L^*/L_0 \langle \varepsilon \rangle)\langle \varepsilon \rangle. \tag{15}$$

Substituting Eq. (15) in the expression for L_0, proportional to L^*, we define the dissipative function of the medium, Eq. (13):

$$\langle D \rangle = k\sqrt{(1-c)(L^*/L_0 \langle \varepsilon \rangle)\langle \varepsilon \rangle} + \\ + \frac{1}{2}\eta(L^*/L_0 \langle \varepsilon \rangle)\langle \varepsilon \rangle + a(L^*/L_0 \langle e \rangle)\langle \varepsilon \rangle. \tag{16}$$

In the case of preferred orientation of the ellipsoids the ratio of the tensors L^*/L_0 may be anisotropic, so that the effective parameters of the medium will also be anisotropic. In the case of isotropic L^* we denote the components of the isotropic tensor L^*/L_0 by $(3f_1(c), 2f_2(c))$, $f_1(c) = \infty$. For an incompressible matrix, L^*/L_0 is independent of the matrix rheological characteristics and is defined solely by the geometric structure of the inclusions. Then the value of L^*/L_0 coincides with the analogous effective quantity of the linear-elastic problem for rigid inclusions in an incompressible matrix [2]. For a macro-isotropic suspension we obtain a dissipative function (1) with effective parameters

$$k^* = k\sqrt{(1-c)f_2(c)},\ \eta^* = \eta_0 f_2(c)^{\frac{n+1}{2}}(1-c)^{\frac{(1-n)}{2}}(\langle э_{ij}\rangle\langle э_{ij}\rangle)^{\frac{n-1}{2}}, \\ a^* = af_2(c) \tag{17}$$

and loading function $\langle\sigma_{ij}{}^a\rangle\langle\sigma_{ij}{}^a\rangle = k^*$ with active stress tensor deviator $\langle\sigma_{ij}{}^a\rangle = \langle\sigma_{ij}\rangle - \delta_{ij}\langle\sigma_{kk}\rangle/3 - \eta^*\langle\varepsilon_{ij}\rangle - a^*\langle e_{ij}\rangle$.

It follows from Eq. (17) that to predict the effective parameters k^*, η^*, a^* it is necessary to know only the rheological data for the matrix and the function $f_2(c)$. The latter can be found not only by calculation, for example, with the effective field method, but also from indirect experiments. For Newtonian suspensions with $n = 1$, $k = a = 0$ it is sufficient to determine the relative change in viscosity η^*/η_0 of the suspensions with the same fractional amount of filler as the composition with non-Newtonian properties, whereupon $f_2(c) = \eta^*/\eta_0$.

Example. For spherical inclusions of a single size the value of $f_2(c)$ was found previously [2]: $f_2(c) = 1 + {}^5/_2\cdot c(1-31c/16)^{-1}$. Figure 1 shows experimental data of [3] for n = 1 and [4] for n = 0.41, constructed in the coordinates $\theta = \ln\{\eta^*[\eta_0(\langle э_{ij}\rangle\langle э_{ij}\rangle)^{\frac{n-1}{2}}]^{-1}\} \sim c$. The data of [3] correspond to a Newtonian suspension of rigid spheres of a single size in water, while the data of [4] describe the nonlinear rheological properties of a suspension of calcium carbonate in fused polypropylene at 200°C. Curves 1-3 were calculated for n = 1 with equations from [5], the present Eq. (17), and from [6]. Curves 4, 5 are for n = 0.41 by the present Eq. (17) and [7] respectively. In the coordinate system used the curves are independent of the parameter η_0. The refinement of the calculation by the effective field method is due to consideration of binary inclusion interaction. For comparison Fig. 2, curves 1, 2 shows $(k^*/k)^2 \sim c$, calculated with Eq. (17) and after [6], independent of the matrix rheological properties.

NOTATION

k, plasticity limit; $\eta(\varepsilon_{ij})$, nonlinear viscosity; a, ordering parameter; σ_{ij}, ε_{ij}, e_{ij}, stress, deformation rate, and accumulated plastic deformation tensors; $X = (V_k, x_k, a^i, \omega_k)$, set of inclusions v_k with characteristic functions V_k, centers x_k, semiaxes a_i, and Euler angle set ω_k; ω, modified deformation rate; G, fundamental solution of Lamet problem; $\varphi(v_m/v_n)$, arbitrary distribution density; $\psi(\omega_m)$, density of distribution ω_m; $\overline{\sigma}$ and $\overline{\sigma}_{1,\dots,n}$, effective fields; x, y, coordinates.

LITERATURE CITED

1. V. A. Buryachenko and A. M. Lipanov, Prikl. Mekh., No. 11, 105-111 (1986).
2. V. A. Buryachenko, Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 3, 69-76 (1987).
3. I. M. Krieger, Adv. Coll. Interf. Sci., No. 3, 111-136 (1972).
4. C. D. Han, J. Appl. Pol. Sci., 18, 821-829 (1974).
5. Yu. A. Buevich and A. Yu. Zubarev, Inzh.-Fiz. Zh., 50, No. 4, 434-445 (1984).
6. V. D. Dudakalenko and S. P. Shapovalov, Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 142-145 (1984).
7. V. J. Kremesec and J. C. Slattery, J. Rheolog., 21, No. 4, 469-471 (1977).